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KENDALL ANALYSIS OF CANNON PRESSURE VESSELS

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The work of the late U.S. Army Benét Laboratories engineer David P. Kendall encompassed a broad range of design and analysis of high pressure vessels for use as cannons. He used classic results from research in engineering mechanics to develop descriptive expressions for limiting pressure, autofrettage residual stresses and fatigue life of cannon pressure vessels. Three types of analysis are described in this paper to validate Kendall's design procedures: 1) Yield pressure tests of cannon sections compared with a yield pressure expression; 2) Autofrettage hoop residual stress measurements by neutron diffraction in cannon sections compared with expressions; 3) Fatigue life tests of cannons following proving ground firing and subsequent laboratory simulated firing compared with Paris-based fatigue life expressions that include post-test metallographic determination of the initial crack size due to firing.

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ABSTRACT

Engineering mechanics analysis of cannon pressure vessels is described with special emphasis on the work of the late U.S. Army Benét Laboratories engineer David P. Kendall. His work encompassed a broad range of design and analysis of high pressure vessels for use as cannons, including analysis of the limiting yield pressure for vessels, the autofrettage process applied to thick vessels, and the fatigue life of autofrettaged cannon vessels. Mr. Kendall's work has become the standard approach used to analyze the structural integrity of cannon pressure vessels at the U.S. Army Benét Laboratories.

The methods used by Kendall in analysis of pressure vessels were simple and direct. He used classic results from research in engineering mechanics to develop descriptive expressions for limiting pressure, autofrettage residual stresses and fatigue life of cannon pressure vessels. Then he checked the expressions against the results of full-scale cannon pressure vessel tests in the proving grounds and the laboratory. Three types of analysis are described here to validate Kendall's design procedures: [i] Yield pressure tests of cannon sections compared with a yield pressure expression, including in the comparison post-test yield strength measurements from appropriate locations of the cannon sections; [ii] Autofrettage hoop residual stress measurements by neutron diffraction in cannon sections compared with expressions, including Bauschinger corrections in the expressions to account for the reduction in compressive yield strength near the bore of an autofrettaged vessel; [iii] Fatigue life tests of cannons following proving ground firing and subsequent laboratory simulated firing compared with Paris-based fatigue life expressions that include post-test metallographic determination of the initial crack size due to firing. Procedures are proposed for Paris life calculations for bore-initiated fatigue affected by crack-face pressure and notch-initiated cracking in which notch tip stresses are significantly above the material yield strength.

The expressions developed by Kendall and compared with full-scale cannon pressure vessel tests provide useful first-order design and safety checks for pressure vessels, to be followed by further engineering analysis and service simulation testing as appropriate for the application. Expressions are summarized that are intended for initial design calculations of yield pressure, autofrettage stresses and fatigue life for pressure vessels. Example calculations with these expressions are described for a hypothetical pressure vessel.

INTRODUCTION

The technical accomplishments of the late U.S. Army Benét Laboratories engineer David P. Kendall can be divided into three areas of pressure vessel technology: yield pressure, autofrettage residual stresses, and fatigue life. These three areas of pressure vessel analysis addressed by Kendall will make up the three sections of results described here. In each section the derivation of expressions that Kendall based on research in engineering mechanics will be briefly described, followed by comparisons with test results taken directly from cannon firings and laboratory simulation of firing. References [1-3] are example publications of Kendall's work; additional references will be listed as the results are described. Figure 1 shows the nominal vessel configuration and nomenclature considered here.

It is significant that Kendall’s body of work led *directly* to the current approach to structural integrity of cannon pressure vessels at the U.S. Army Benét Laboratories. This is clearly supported by the forthcoming close comparisons between Kendall’s analysis and structural integrity results from cannon pressure vessels subjected to yield pressure testing, autofrettage procedures and to firing and simulated firing fatigue loading conditions. In addition, Mr. Kendall’s work has been adopted by various organizations involved with the design and manufacture of high pressure vessels and by professional organizations that develop design codes for pressure vessels. His concepts and approaches have underpinned the analyses and experimental work of a major international technical cooperation program on gun design. This program included leading researchers from government organizations and universities within the United States, United Kingdom, Canada, and Australia, and effectively defined current gun design methodologies.

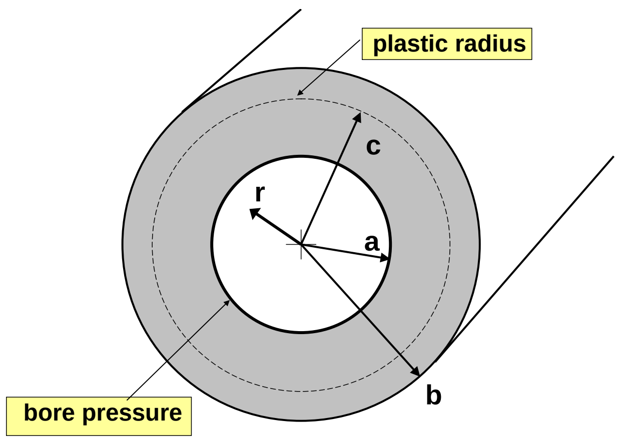

Figure 1: Autofrettaged cannon with plastic radius, *c*.

YIELD PRESSURE

A basic requirement of any pressure vessel is ability to sustain an applied pressure with no immediate damage. For a pressure vessel such a limit pressure is typically that which results in no measurable permanent yielding deformation, or perhaps an allowably small permanent yielding. A colleague of Kendall, Anthony P. Parker, has recently described the type of yield pressure expression [4] that Kendall used for autofrettaged vessels. This yield pressure is the internal pressure, $P_{y\epsilon}$, that was initially required to drive the plastic radius out to a given position, c, during autofrettage loading for plane strain conditions, see Equation (1).

$$P_{y\epsilon} = [2/3^{1/2}]\, Sy\, [\ln(c/a) + (b^2 - a^2)/2b^2] \qquad (1)$$

Note in the expression that yield pressure varies directly with the yield strength of the vessel material, Sy . For this reason Kendall suggested that yield pressure is best determined using a yield strength input to Equation (1) measured from a near-bore location and hoop orientation of the vessel of interest. This type of measurement accounts for any strength variations due to differences in forging and heat treatment at different vessel locations. Thus the best measure of yield pressure requires the post-test cutting of the vessel to allow measurement of the material yield strength at the bore location and hoop orientation that most directly control yielding of a pressure vessel.

The $[2/3^{1/2}]$ factor in Equation (1) is for ideal plane strain conditions, while for plane stress conditions this factor is 1.0 [4]. So the yield pressure of vessels should be well predicted with a factor between 1.15 and 1.0 in Equation (1). The question arises as to where in this 15% range do yield pressures of typical cannon pressure vessels lay. The following comparison of measured yield pressure, P_m, from cannon pressure vessels with calculations from Equation (1) addresses this question, see Table 1.

Pressure vessel results are shown in Table 1 for breech sections of four ASTM A723 steel cannon tubes that were pressurized in small increments, with strain gages used to monitor permanent OD hoop strain after each increment [5]. The measured yield pressure, P_m, is that corresponding to a small but measurable permanent OD hoop strain of 0.01%. This procedure, of a measurement at a small permanent strain, is directly analogous to the 0.1% offset strain typically used in measurement of a material's yield strength. After the tests, tensile yield strength measurements were made from samples cut near the ID in hoop orientation, as Kendall suggested. The first three columns of Table 1 show clearly how directly the vessel yield pressure depends on the appropriately measured yield strength; the ratio of yield pressure to yield strength is nearly constant for four vessels with a wide range of yield strength.

Measured Yield Strength ***Sy*** ; MPa	Measured Yield Pressure ***Pm*** ; MPa	Ratio ***Pm/Sy*** (--)	Equation 1 Yield Pressure ***Pyε*** ; MPa	Equation 2 Yield Pressure ***Py*** ; MPa
1022	637	0.62	670	641
1105	692	0.63	724	693
1177	741	0.63	772	738
1394	875	0.63	914	874

Table 1: Comparisons of measured and calculated yield pressures; for vessels with a=79, b=148 mm; c=114 mm; degree of autofrettage, (c-a)/(b-a) = 51%.

The additional results in Table 1 show calculated yield pressure for comparison with measured yield pressure, to address the earlier question of real vessel yield behavior. The ideal plane strain calculation of yield pressure from Equation (1) gives results 4% to 5% above measured values. This is consistent with real vessel yield behavior being between the plane strain upper limit and the plane stress lower limit. A vessel yield pressure expression that fits between two limits, taking a pragmatic Kendall approach, is the following.

$$P_y = [1.10]\, Sy\, [\ln(c/a) + (b^2 - a^2)/2b^2 \qquad (2)$$

The *[1.10]* factor gives close agreement between measured and calculated yield pressures and is between the 1.00 and 1.15 limits. Note, however, that the yield pressure measurements, although for a wide range of yield strength, are for a single OD to ID ratio of 1.87 and degree of autofrettage $(c-a)/(b-a) = 0.51$. For A723 steel pressure vessels near this configuration and degree of autofrettage, Equation (2) is believed to give a useful estimate of the yield pressure required to produce a small measurable permanent deformation. For vessels of different configuration and autofrettage, yield pressure measurements using near-bore hoop-orientation yield strength measurements are suggested, to verify that Equation (2) continues to give an accurate estimate of yield pressure.

An indication of the effect of extent of autofrettage on the calculated yield pressure can be obtained from Equation (2). This is shown in Table 2 for the a and b dimensions and yield strength of one of the vessels described in Table 1. It is clear that, in addition to yield strength, extent of autofrettage has a significant controlling effect on yield pressure. Autofrettage stresses are considered next.

Extent of Autofrettage *(c-a)/(b-a)* (--)	Yield Pressure *Py* MPa
0%	435
50%	691
100%	763

Table 2: Calculated yield pressure for varying extent of autofrettage; for a=79 mm, b=148 mm, Sy=1105 MPa.

AUTOFRETTAGE STRESSES

A second requirement for many pressure vessels is autofrettage, because the resulting near-bore compressive hoop stresses so improve the vessel function that autofrettage is considered a necessity. And, as with yield pressure, Parker's recent summary [4] provides the basic mechanics expressions used by Kendall to describe autofrettage hoop residual stresses for an ideal elastic-perfectly-plastic vessel. Given that the strain hardening behavior of pressure vessel steels is limited and quite similar to elastic-perfectly plastic, this approach can be used. The expression for the hoop residual stress, S_{hP} in the plastic inner portion of an elastic-plastic, autofrettaged tube under plane strain, von Mises yield conditions is:

$$S_{hP} / S_y = [(2/3^{1/2}] \times [(c^2+b^2)/2b^2 + ln(r/c) - (a^2/\{b^2-a^2\}) \times (1+b^2/r^2)] \times [\{b^2-c^2\}/2b^2+\ln\{c/a\}] \quad (3)$$

where r is the radial position in the tube wall and the other terms have been defined. The hoop residual stress in the remaining elastic outer portion of the tube is:

$$S_{hE} = p_E \ [a^2(b^2+r^2)] / [r^2 (b^2-a^2)] \quad (4)$$

where p_E is pressure that gives Lame stress (Eq. 4) equal to residual stress (Eq. 3) at the plastic radius, *c*. Kendall knew that Equation 3 needed modification to account for the Bauschinger reduction in compressive yield strength in the near-bore vessel material, caused by tensile plastic deformation of this material that occurred during autofrettage. His approach was simply to limit the near-bore hoop compressive stress to 70% of the yield strength. This was easily done here with spread sheet calculations, followed by a slight shift in the limited curve to insure equal areas above and below the zero stress axis, as required for equilibrium. The resulting plots of Equations (3) and (4) are shown as Figure 2, compared with neutron diffraction measurements of hoop residual stress from two sections of an autofrettaged tube [5]. See also Table 3 for dimensions and results from the tube sections. The sections were from different axial locations of the tube, with different outer diameters during autofrettage and thus different resulting plastic radii, *c*, as shown in Table 2. The *a* and *b* values shown are the final machined dimensions [slightly different from the autofrettage dimensions], resulting in a bit under 50% autofrettage for each final section. Figure 2 results for the a=60mm section are for ideal autofrettage [dashed curve] and for Bauschinger affected autofrettage [solid curve] with the 70% of *Sy* limit applied near the bore. The a=79mm results are for ideal elastic-plastic autofrettage, because the ideal autofrettage stresses even at the bore are less than 70% of *Sy*. See again Table 3.

The most significant aspect of the comparison in Figure 2 is just how well the elastic-plastic calculations agree with the neutron diffraction measurements, often regarded as the most reliable type of residual stress measurements. The agreement is good for both the near-bore plastic region and the near-OD elastic region, believed due, as noted earlier, to the similarity of the yielding behavior of pressure vessel steels to ideal elastic- plastic yielding. Note also in Figure 2 that Kendall's 70% concept of Bauschinger limited near-bore residual stresses agrees well with the neutron diffraction results. The -721 MPa limit in the BEF-affected curve coincides quite well with the "tail-off" of the diffraction results, even if this simple limit does not match the shape of the tail-off.

It is also significant that the simple Kendall description of Bauschinger effects on autofrettage residual stress makes some important predictions. His 70% concept applied to the a = 60 mm calculations predicts [in Table 3] that the ID hoop compression is reduced from -929 to -721 MPa. As will be seen in upcoming fatigue life results, this reduction in hoop compression will have a significant effect on fatigue life. Kendall's 70% Bauschinger correction also predicts the configurations for which the Bauschinger reduction of near-bore residual stress is expected to have no effect on vessel performance. The 70% concept applied to the a=79mm calculations predict no change in ID hoop compressive stress, due in this case to the lower diameter ratio of this configuration, even though the extent of autofrettage is nearly the same for the two sections. Simply put, for a case where the near-bore ideal elastic-plastic hoop residual stress does not exceed 70% of yield, there will be no Bauschinger reduction of stress and thus no Bauschinger effect on vessel performance. The vessel diameter ratio, *b/a*, provides a useful indicator of potential Bauschinger effects on performance, given a constant extent of autofrettage. For the calculations summarized in Table 3 those for a *b/a*=2.25 have a 22% Bauschinger reduction in hoop residual stress. Whereas for the calculations for *b/a*=1.94, there is no Bauschinger effect on residual stress or on fatigue life.

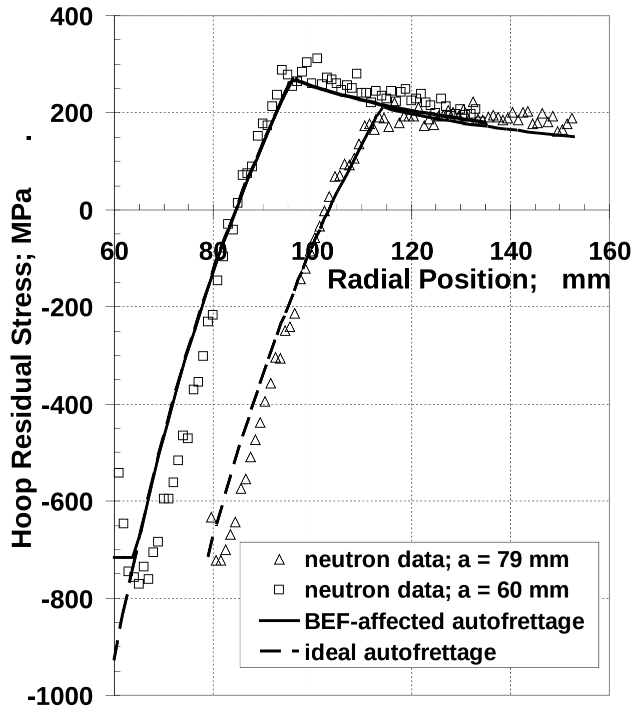

Figure 2: Measured and calculated hoop residual stresses for two sections of autofrettaged 1022 yield strength tube.

Inner Radius	Outer Radius	Yield Radius	Extent of Autofrettage	Diameter Ratio	ID Hoop Stress with BEF	no BEF
a ; mm	***b*** ; mm	***c*** ; mm	***(c-a)/(b-a)***	***b/c;*** (--)	MPa	MPa
60	135	96	48%	2.25	-721	-929
79	153	114	47%	1.94	-715	-715

Table 3: Dimensions and autofrettage residual stresses for two sections of 1022 MPa yield strength tube.

The calculated hoop residual stresses for an autofrettaged A723 steel pressure vessel using Kendall's approach are seen, in Figure 2, to agree well with a quite independent and reliable measurement of residual stresses. And predictions made from the results as to vessel performance seem reasonable. For these reasons the Kendall analysis of autofrettage stresses will next be used for calculations of pressure vessel fatigue life and comparisons with measurements of vessel fatigue life.

FATIGUE LIFE

The classic Paris approach to fatigue life determination has been well described in the technical literature; a summary [6] for the purpose here is shown in Equations (5) and (6). Equation (5) is the usual Paris law for crack growth per cycle, *da/dN*, in terms of stress intensity factor range, ΔK and Paris constants *C* and *m*.

Equation (6) is the final integrated form of Equation (5) where the cycles to failure, N, depends on: the preexisting initial crack length, a_i; the final crack length [for a vessel], $(b-a)$; the Paris constants C and m; the crack shape factor, f varying between 1.12 for a straight-fronted edge crack and 0.66 for a semicircular edge crack [7]; the applied and residual stresses, $S_{applied}$ and $S_{residual}$ combined with any stress concentration, k_t that applies; and vessel internal pressure, p.

$$da/dN = C \Delta K^m \qquad (5)$$

$$N = [a_i^{(1-m/2)} - (b-a)^{(1-m/2)}] \;/\; [C\, f^{\,m}\, \pi^{m/2}\, (m/2 - 1)\, (k_t\, S_{applied} + k_t\, S_{residual} + p)^{\,m}]$$

$$\textbf{\textit{for }} \boldsymbol{S_{residual} < 0} \qquad (6)$$

Determining the inputs to Equation (6) requires care, because small changes in inputs can produce large changes in life. Initial crack length, a_i, has significant control on life and should be measured by destructive metallographic methods when possible. C and m should be measured by prescribed laboratory methods [8]. The combination of stresses that make up the effective stress range is particularly critical; Kendall helped develop the following approach to determining the stress range for Paris calculations. The $S_{applied}$ from the usual Lame expressions for a pressurized vessel is the basis for the Paris stress range, with the following modifications. First, if the pressure is applied to the crack faces in addition to the vessel ID, as is the case for a crack originating from the ID, then pressure, p, is added to $S_{applied}$, as shown in Equation (6). The second modification to Paris stress range is that due to $S_{residual}$, where Kendall suggested accounting for near-bore autofrettage compressive stresses, as shown in Equation (6), but ignoring near-OD autofrettage tensile stresses. The rationale for ignoring tensile residual stresses is that the combination of applied Lame tensile stresses and tensile residual stresses often leads to yielding near the vessel OD. And yielding is particularly likely at an OD notch, which is the source of most vessel OD fatigue cracking. The notch-tip yielding is believed to overwhelm the pre-existing notch-tip tensile residual stresses by mechanical stress relaxation near the OD notch, so that only the applied stresses remain to control cracking.

A comparison of Paris calculations of life with measured lives from cannon pressure vessels is shown in Table 4. This is the same type of comparison made by Kendall, given that he was a principal developer of the autofrettage and fatigue test procedures for this cannon pressure vessel. The measured bore-initiated lives were from the chamber section of six ASTM A723 steel cannon pressure vessels with a mean yield strength of 1140 MPa. This section of the vessel [similar to that described in Figure 2] was believed to have about a 50% extent of autofrettage, based upon Kendall's analysis of permanent deformation of the vessel as the result of autofrettage. The recent neutron diffraction results discussed earlier in relation to Figure 2 verify the earlier work. The Paris constants used for the Table 4 calculations were measured recently [8] from samples cut from this type of vessel. The initial crack depth, ai, for these vessels was also well verified, this being a key reason for using these results for comparison with Paris calculations. The ID of the chamber section of these vessels is electro-plated with chromium to a consistent 0.10 mm thickness, and the chrome plate cracks [due to thermal shock] following the first few firing cycles, producing an initial crack of a known 0.10 mm depth.

Measured Lives		$S_{applied}$	$S_{residual}$	N_{Paris}
cycles		MPa	MPa	cycles
range:	4,649 - 8,204	1258	-678	6,900
mean:	6,140			

Table 4: Comparison of measured ID fatigue lives with Paris calculations; yield strength = 1140 MPa, *a* = 79 mm, *b* = 148 mm, *(c-a)/(b-a)* = 0.51, Paris constants *C* = 1.85E-11, *m* = 2.78, *ai* = 0.10 mm, pressure, *p* = 70 MPa.

Continuing with the description of the inputs to fatigue life calculation [Equation (6)], the crack shape factor, *f*, is 0.66, that for a semicircular crack believed to be present during the small-crack growth portion of the life that has predominant control of life. Growth of small semi-circular cracks is expected from the scattered points of chrome plate cracking. In this case, as in most, the tests were not interrupted in order to directly determine the early crack shape. Regarding the stress related inputs to life [the last bracketed term in Equation (6)], $k_t = 1$ because there is no stress concentrator at the bore, and the pressure applied to the bore and the crack faces is *p* = 700 MPa. The applied and residual hoop stresses at the bore, $S_{applied}$ = 1258 MPa and $S_{residual}$ = -678 MPa listed in Table 4, were from the well known [4] Lame expressions and from Equation (3), respectively.

With the various inputs to the Paris calculation of fatigue life of Equation (6), the result shown in Table 4 is 6,900 cycles for the conditions of the six vessel tests, well within the range of measured lives from the tests. Based on this good agreement, the approach as summarized will be used for further Paris life comparisons. Table 5 shows the important effect of extent of autofrettage on fatigue life for the conditions of the fatigue tests described in Table 4. As expected autofrettage gives a significant extension of fatigue life, and there is a “diminishing return” in the amount of life extension for large amounts of autofrettage, with only a 34% increase in predicted life for an increase in autofrettage from 50% to 100%.

The final section of results is a descriptive example of the various methods of analysis pioneered by Kendall, as applied to a hypothetical pressure vessel.

Extent of Autofrettage	Fatigue Life
(c-a)/(b-a)	N_{Paris}
(--)	cycles
0%	2,100
50%	6,800
100%	9,100

Table 5: Paris life calculations at ID for various amounts of autofrettage; yield strength = 1140 MPa, a = 79 mm, b = 148 mm, Paris constants C = 1.85E-11, m = 2.78, ai = 0.10 mm, ID pressure, p = 700 MPa, f = 0.66.

EXAMPLE VESSEL CALCULATIONS

Consider an example autofrettaged vessel made from 1000 MPa yield strength ASTM A723 pressure vessel steel, with a=40mm, b=100mm, containing a pressurized, small diameter, radial through-hole. Cannons sometimes require such holes. Design estimates can be made for the vessel for: yield pressure from Equation (2); autofrettage stresses from Equations (3) and (4), including the near-bore Bauschinger effect based on 70% of yield strength; and fatigue life from Equation (6). These Kendall methods for describing the structural integrity of a pressure vessel can certainly not replace service simulation fatigue life tests or finite element analysis, particularly for critical applications. However they can provide an initial analysis and a "sanity check" for more comprehensive tests and analysis.

Extent of Autofrettage	Yield Pressure	Calculated Life; Bore-Initiated		Calculated Life; Hole-Initiated	
		no BEF	with BEF	location	life
(c-a)/(b-a)	***Py-2*** ; MPa	***N*** ; cycles	***N*** ; cycles	***r*** ; mm	***N*** ; cycles
0%	460	6,900	6,900	40	820
50%	900	69,800	32,300	60	3,300
100%	1010	215,000	34,300	67	4,200

Table 6: Calculated yield pressures and fatigue lives for an example pressure vessel; yield strength = 1000 MPa, a = 40 mm, b = 100 mm, Paris constants C = 1.85E-11, m = 2.78, ai = 0.01 mm, applied pressure, p = 700 MPa, f = 0.71.

Table 6 summarizes some key results for the example vessel for three amounts of autofrettage. The calculated yield pressures for the vessel [Eq. 2] vary considerably with amount of autofrettage, as expected for this example vessel with a high b/a = 2.5. Note however that there is a diminishing return on the increase in yield pressure with increasing autofrettage; an increase from 50% to 100% autofrettage resulted in only a 12% increase in yield pressure.

The hoop residual stresses [Eq. 3 and 4] were calculated for the three amounts of autofrettage; results for 50% autofrettage are shown as Figure (3). The most significant Bauschinger effect is near the bore, as expected due to the limitation of near-bore compressive residual stress to 70% of the yield strength. As seen in Table 6, this near-bore limitation has its effect on fatigue life. The *bore-initiated* fatigue lives were calculated and listed in Table 6 [using Eq. 6; k_t=1; $S_{applied}$ from Lame] for the same 700 MPa internal pressure as with the previously described tests and for the lower bound initial crack size of pressure vessel steel, *ai* = 0.01 mm. Note that for 50% autofrettage the life including Bauschinger effect is less than half of that with no Bauschinger effect, and for 100% autofrettage the life including Bauschinger effect is less than a sixth of that with no Bauschinger effect.

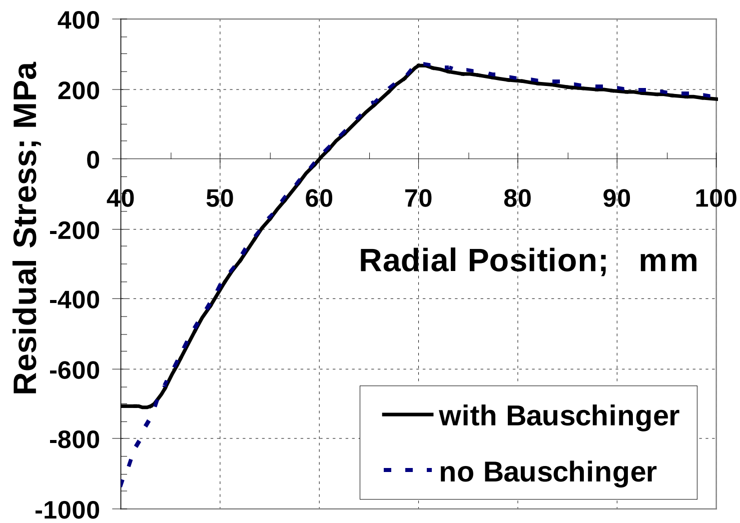

Figure 3: Calculated hoop residual stresses for 1000 MPa yield strength example vessel; *a* = 40 mm, *b* = 100 mm, Paris constants *C* = 1.85E-11, *m* = 2.78, *ai* = 0.01 mm, applied pressure, *p* = 700 MPa.

Recalling that a pressurized radial through hole is present in the example vessel, it may be of interest to compare the *hole-initiated* fatigue lives that are predicted at the hole ID with the *bore-initiated* fatigue lives, just discussed. The hole-initiated lives are calculated using Equation (6), as before, but with some different stress inputs. The applied stress range, $S_{applied}$ is as before, from the Lame expression. The k_t=3 for both applied and residual hoop stress, given that the hole diameter is small compared to vessel wall thickness, so the full stress concentration effect of the hole is felt near the hole. The residual hoop stresses are calculated from Equation (3), and, as before, only the compressive hoop residual stresses are used in the Equation (6) calculation of life. The rationale for using compressive stresses only is as before; the tensile hoop residual stresses, when combined with the applied Lame tensile stresses, give rise to yielding near the hole, so the effect of the tensile hoop residual stresses is lost. For example, for the entire outer tensile-residual-stress region of the 50% autofrettaged vessel, the applied plus residual hoop stress at the hole stress concentration is more than twice the yield strength. So yielding is inevitable, and tensile residual stresses have no effect on life as calculated here.

The results of the calculations of hole-initiated fatigue life are shown in the last section of Table 6 and in Figure 4, for various amounts of autofrettage. Table 6 shows that, as expected, the case of 0% autofrettage results in a very low fatigue life at the bore radial location. For 50% and 100% autofrettage, the minimum hole fatigue lives are much higher that at the bore, and the radial location for minimum life moves toward the mid-wall location, where the hoop residual stress is near zero. Figure 4 shows the life calculation results for all radial positions of the example vessel for the case of 50% autofrettage. The minimum life of 3,300 cycles at about r = 60 mm is indicated; note that this is the radial position of zero hoop residual stress shown in Figure 3. Outside of the zero-residual-stress point [$r > 60$] the hoop residual stress, being tensile, has no effect on life as calculated here, and life increases with r because the applied stress decreases with r. Inside of the zero-residual-stress point [$r < 60$], the hoop residual stress is increasingly compressive, causing an increase in life. But very near the bore, where the Bauschinger effect limits the hoop compressive residual stress, there is a decrease in fatigue life. Of course the critical fatigue life is the minimum life, that at the zero-residual-stress point for the autofrettaged example vessel. And these minimum lives [in Table 6] for hole-initiated fatigue are barely a tenth of the bore-initiated lives, as would be expected due to the stress concentration of the hole.

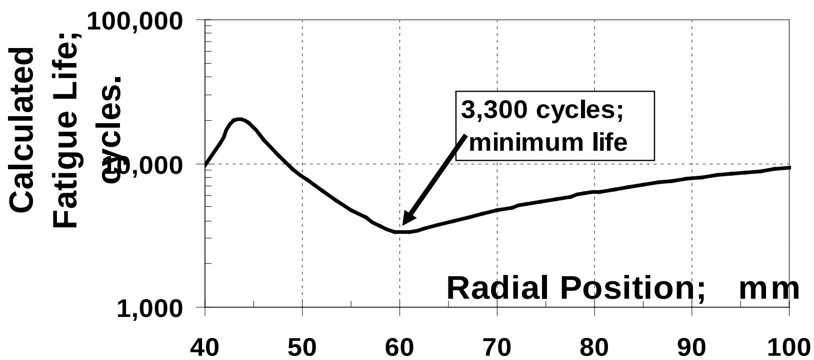

Figure 4: Calculated fatigue life for 1000 MPa yield strength example vessel; a = 40 mm, b = 100 mm, $(c-a)/(b-a)$ = 0.5, Paris constants C = 1.85E-11, m = 2.78, ai = 0.01 mm, applied pressure, p = 700 MPa.

SUMMARY

Engineering mechanics expressions derived from the research literature were suggested by Kendall as initial design methods for autofrettaged pressure vessels, to be followed by further tests and analysis as needed.

[i] *An expression for the yield pressure* of an autofrettaged pressure vessel [Eq. (2)] was proposed for materials that approximate elastic-perfectly-plastic yield behavior, including many pressure vessel steels. The expression is shown to be in close agreement with yield pressure measurements from cannon pressure vessels with 1000-1400 MPa yield strength A723 steel, diameter ratio of about 2.0, and 50% extent of autofrettage. The yield pressure calculations require yield strength measurements from samples near the vessel bore and in the vessel hoop orientation.

[ii] *Expressions for hoop residual stress* resulting from autofrettage of a pressure vessel [Eq. (3) and (4)] were proposed by Kendall with a modification to account for the Bauschinger reduction in near-bore compressive strength resulting from autofrettage. The Bauschinger modification limits the near-bore hoop compressive residual stresses from elastic-perfectly-plastic analysis to 70% of the material yield strength. These expressions with the 70% limitation are shown to be a close match to recent hoop residual stress measurements from neutron diffraction results, for both the plastic-deformation inner region and the elastic-deformation outer region of a partially autofrettaged vessel. The near-bore Bauschinger affected neutron measurements were also approximately matched by the 70% limit.

[iii] *An expression for Paris fatigue life* for an autofrettaged pressure vessel [(Eq. (6)] is proposed that includes modifications suggested by Kendall. The key modification is the above-mentioned 70% limitation to near-bore hoop compressive residual stress, because even modest changes in near-bore stresses have large effects on fatigue life. A Paris calculation of life is well within the range of measured lives from six cannon pressure vessel fatigue tests, for which initial crack size was known accurately from post-test metallographic results and the Paris fatigue crack growth constants were carefully measured.

Another modification of life calculations derived from Kendall's work involved the effect of tensile residual stresses, typical of outer regions of a vessel, particularly when a stress concentration is present. Calculations for a through-hole stress concentration in a vessel are described that compare Paris lives at various hole locations and amounts of autofrettage. The hole location with minimum life is near mid-wall of the autofrettaged vessel, where the hoop residual stresses are zero. Toward the bore from the zero-hoop-stress point the fatigue life is higher than the minimum, because the compressive residual hoop autofrettage stresses counteract the applied tension. Toward the OD from the zero-hoop-stress point the fatigue life is also higher, because the tensile hoop residual stresses are eliminated by yielding near the hole; the combination of residual and applied tensile hoop stress and the hole stress concentration gives rise to stresses well above yield. And the loss of tensile residual stresses by yielding results in a higher fatigue life.

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